LIBRARIES AND THE OTHER TRIANGLE UNDER THE DEMAND CURVE

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In this note a simple theory of library services is presented. The theory was developed as a normative tool to aid libraries in answering the question: which books should be bought by the library? Although developed for normative purposes, the theory generates testable predictions. In future work I shall try to test them. Further, the theory should prove applicable to a number of public services (e.g., neighborhood health centers and swimming pools) where the individual has the option to make or buy a similar service on the market.

If we could measure the aggregate demand curve for library books, it would be a relatively simple task to determine which books the library should buy. Ignoring distributional considerations, it should buy those books whose appropriately discounted benefit stream over the life of the book is greatest, where benefit in any period can be approximately measured by the area under the demand curve. That is, suppose q_L is the quantity of a particular library book demanded in a given period. It is a function of the price of using the library for this book p_L, the price of the book p_B, and x, a vector of other variables,

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$$(1) \qquad q_L = f(p_L, p_B, x) \ .$$

There are certain complications such as possible non-price rationing of the library book, but ignoring those complications we could, if we knew the demand curve, measure benefit in the usual fashion, as

$$\int_0^{\bar{q}_L} f^{-1} \, dq_L \ ,$$

where $\bar{q}_L$ is the quantity demanded at the existing price $\bar{p}_L$.

Unfortunately such a demand curve is very hard to measure because preferences are never revealed in the marketplace. This, of course, is the usual problem for publicly supplied goods. The point of this paper is that the benefit from publicly provided goods which are close substitutes for goods that can be purchased on the market can be approximately measured by using the demand curve for the private good, a demand curve which not only is measurable but frequently is not difficult to measure.

Suppose we write the demand curve for a book:

$$(2) \qquad q_B = g(p_L, p_B, y)$$

where q_B = quantity of book demanded

p_L = price of library book

p_B = price of book

y = vector of other variables.

Such a demand curve is shown in Fig. 1 (p. 3). Suppose for now that p_L equals zero; $\bar{p}_B$ and $\bar{q}_B$ are the price and quantity of the book actually bought. Then ignoring time and transaction costs, the shaded area ABC

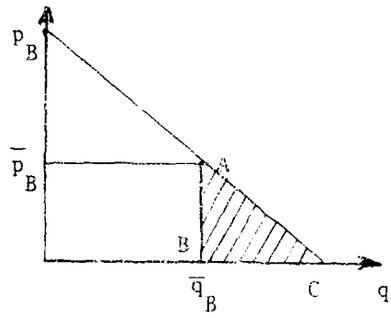

Fig. 1

can be taken as a probable upper bound on the benefit that could be provided the community by having the book available at the library. The area ABC measures the amount which those individuals who are excluded by the price are willing to pay to own the book. Presumably this is the maximum amount these individuals are willing to pay to borrow the book from the library.* Note that, cet. par., the shaded area is larger: 1) the more elastic the demand curve below $\bar{p}_B$; 2) the greater $\bar{p}_B$. The area is not necessarily increased by a shift outward in the demand curve.

In general, if we assume y and p_L fixed and write the demand curve $q_B = f(p_B)$, the area we are interested in (which we call B for benefit) equals

$$B = \int_0^{\bar{p}_B} f(p_B)\, dp_B - \bar{p}_B\, \bar{q}_B .$$

*The idea of the triangle under the private demand curve's providing an upper bound also carries over to swimming pools and neighborhood health centers. Individuals are presumably willing to pay a premium for having their private pool or private physician.

The case of a linear demand curve is particularly simple. Let $q_B = a - bp_B$, where a and b are positive constants. Then it is easy to verify that $B = b(\bar{p}_B)^2/2$, so that B is independent of a, the intercept term. B rises proportionally with b, the slope of the demand curve and as the square of $\bar{p}_B$, the price of the book.

Before setting out a decision rule for libraries, we must take account of some qualifications. First, we assumed that use of the library entailed no costs. Trips to the library do cost something in time and money however. If this cost were the same for all individuals and for all books, we could make it the lower limit of integration in the integral used to define B. Since individuals have different values for time, however, the cost is not the same for all individuals. Likewise, the marginal cost of obtaining a second book from the library is small if one is making a trip for another book. Neither of these considerations should much affect the theory, however. So long as the tastes of individuals in books are not correlated with the cost to them of using the library or with propensities to check out more than one book, the effect of transaction costs is like a lump-sum tax on the total benefit the library is providing the community. It should not affect the relative desirability of various books and periodicals. This is also true of any change in x, the vector of other variables affecting demand for library services.

Second, there is the problem of time. Time enters in two ways. Not all those who demand the book can use it at the same time. If individuals keep a book they check out an average of two weeks, a maximum of twenty-six individuals can use the book during the year.

Suppose fifty-two individuals wish to read the book. In that case, a discount factor should be applied to the benefits supplied to those individuals whose reading is deferred. There is more to the problem than that, however. Time will also affect what an individual will pay to have the services of the book. Some books have a fad value; the value I place on reading last year's best seller may be much lower this year. Further, even if I read a book this year, I may place some value on having it available next year. Although some books depreciate very rapidly after reading, others do so much more slowly. Thus, my demand for the services of a book in future time periods will depend upon: a) whether or not I have read the book; b) what type of book it is.

When considering benefit supplied to the community, the library should clearly take into account the (discounted) value the book will provide in future periods. However, the benefit provided in any period should only be considered as accruing those who actually use the book during the period and those who would pay for having the services of the book available even if they do not use it.* Also, the benefit should be the amount they could be expected to pay in that period, not in the present time period.**

With these factors in mind and ignoring distributional considerations we can state the (reasonably obvious) decision rule: Rank all books in order of net benefit (benefit less cost of book). With a

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*Burton Weisbrod, "Collective-Consumption Services of Individual-Consumption Goods," Quarterly Journal of Economics, 78, August 1964, pp. 471-477.

**We ignore problems related to uncertainty in making this decision.

fixed budget, buy as many books as possible beginning with those with the largest net benefit so long as net benefit is positive. With no fixed budget, buy all books with a positive net benefit. In both cases transaction costs should be deducted in the calculation of net benefit. (Note, however, that the theory cannot yield an answer to the question: How large should the library budget be? The answer to that question depends upon the opportunity cost of public sector funds.)

The effect of this decision rule is that the library will tend to buy books which are expensive and which have a high price elasticity. To be precise about "expensive," in general as price rises, the size of triangle ABC will increase rapidly. For linear demand curves, the increase will always be greater than the increase in p up to the point where the demand curve intersects the price axis (the demand curve would have to be reasonably concave for this not to be true); thus, cet. par., net benefit will be greater for higher priced books. In practical terms, this means the library should tend to buy reference books (though not necessarily textbooks, since demand for textbooks will usually be inelastic) and other books for which a number of people are willing to pay something less than the purchase price; it should not necessarily buy very popular books (a shift in the demand curve may not increase the size of triangle ABC); also it should tend not to buy books which will later be published in paperback (when the paperback edition comes out, the size of triangle ABC will diminish).

Some other prescriptions that may be drawn from this model are: 1) Ignoring distributional considerations (as we shall continue to do), the library can increase welfare by charging a fee to reserve a book. By charging such a fee, the library will increase the probability that those who place the greatest value on the book will read it first; thus, the larger discount factors will be applied to the lesser benefits.* Also, the possibility of reservation by paying a fee reduces the probability of socially inefficient trips or calls to the library to obtain or inquire about a book that is not in; 2) the library should not simply accumulate books, but should consider selling off some older books; by using the proceeds from the sale to buy new books, it may be able to increase the community's welfare; 3) except for the above mentioned reserve charges, prices to use the library should be low or zero; like a collective good with a zero marginal cost, a positive price which excludes individuals creates a deadweight loss; insofar as there are positive marginal administrative costs in issuing cards and loaning books, however, a fee is justified; 4) fines for not returning books on time should reflect demand for the book; in particular, theoretically, no fine should be imposed for a book which is not demanded. Two qualifications of this rule are in order: 1) it is difficult to know that a book is not demanded; thus some small fine for lateness may be justified by the presumption that any book on the shelf might be demanded; 2) keeping a book longer may raise the probability of loss. Fines should, however, be higher on

*Since the value of time differs among people, those who place the greatest value on the book will not necessarily read it first.

books which are known to be demanded.

Before setting out positive implications of the theory, we should take note of a probable relationship between price, quantity, and demand elasticity.

Publishers of a book have a legal monopoly to publish the book; therefore, pricing of books becomes a straightforward exercise in the theory of monopoly pricing.* For simple monopoly, that theory, as is well known, predicts that $\hat{p}$, the profit maximizing price, will equal $MC/(1 - 1/e)$, where MC is marginal cost and e is the price elasticity of demand. For constant marginal cost (probably a reasonable approximation) $d\hat{p}/de$ equals minus MC $(e^{-2})/(1 - e^{-1})^2$, which is negative. Hence, the lower the price elasticity of demand, the higher $\hat{p}$. Thus, the two factors in a library's decision to purchase a book -- price and elasticity -- can often be predicted to work at cross-purposes. (This sheds light on why textbooks may be relatively expensive but rather poor candidates for library purchase.)

However, there is also the possibility that the book will be brought out in paperback. The possibility of paperback publication means that the theory of discriminating monopoly rather than the theory of simple monopoly is applicable. In this case, cross-elasticities between paperback and hardback editions become important in pricing. However, it can be shown that as price responsiveness (- quantity demanded/- own price) increases, cet. par., the profit-maximizing price

*It is interesting to note that this may be one reason for the existence of a library; that is, its existence helps reduce the deadweight loss from a price above marginal cost.

for that product will usually fall.* (The profit-maximizing price for the other product, if price responsiveness for the first product increases, may rise or fall.) Hence, a more realistic picture of the market than simple monopoly does not change the conclusion of the previous paragraph that price and elasticity will usually be negatively related.

These theories generate testable predictions, and in future work I hope to test them. First, it would be interesting to know whether publishers do price as monopolists. Some survey data on demand functions for books may be available. If so, the hypothesis that as elasticity rises, price falls can be tested. Also, decision rules for bringing out a paperback can be explored, so that a library can predict which books are likely to appear in paperback.

Second, the theory predicts that an individual's demand for a library book relative to purchasing the book will be higher, cet. par., the higher the price of the book. Hence, collective demand for a library book will be higher, cet. par., the higher the price of the book. This could be tested by trying to form homogeneous classes of books (such as novels that have not appeared in paperback) and comparing the ratios of library circulation to book sales for various priced books. It is important to note, however, that if higher price

*It will fall provided $\partial f/\partial p_L \cdot \partial g/\partial p_N - \partial f/\partial p_N \cdot \partial g/\partial p_L > 0$, where f is the demand function for product L, g is the demand function for product N, p_L and p_N are the price of L and N respectively. Otherwise it will rise. In other words, it will fall if the product of the partials with respect to own price exceeds the product of the partials with respect to cross price.

is associated with lower price elasticity of demand, this hypothesis may be rejected. Third, the theory predicts that demand for a library book will fall when a paperback edition is issued. Although the ceteris paribus conditions are again difficult to fulfill, this could be tested by computing the ratio of circulation after issuance of paperback to circulation before issuance for a number of books of one type (e.g., novels). One needs a reference group; this might be derived from a sample of books of the particular type which have not been issued as paperbacks; summing their circulation in each period (e.g., month) after acquisition and dividing by the number of books in the sample yields a time profile of circulation for books which did not appear in paperback. This could be used to derive a ratio to compare with the previously defined ratio for books which appear in paperback.

In this note we have presented both the normative and positive aspects of a theory of book purchase for libraries. Although developed in the context of library services, the theory appears applicable to a number of publicly-provided services which are substitutes or near-substitutes for privately marketed goods. In effect, it uses part of the area under the demand curve for the private good to approximate the area under the (presumably unmeasurable) demand curve for the public good. Thus, it can help make public sector resource allocation more rational.